

DEDICATION

Remembering Paul Turán

Paul Turán died 20 years ago on September 26, 1976, yet his mathematics is still as fresh as when he did it. Primarily this is because Turán frequently looked at problems from a new point of view. I have heard this expressed by those who knew his work in areas I know well as well as in areas I do not understand at all. This often means that it will take decades to decide on the real worth of some of his contributions, so we should not rush to judge that which we still do not completely understand.

In Turán's "Collected Papers," there are two interesting articles on him and his mathematics by Paul Erdős and Gábor Halász. However, neither one comments on an important aspect of Turán's service to mathematics, namely, his editing of the selected papers of Alfréd Rényi and the collected papers of Leopold Fejér. Here is a little on why this was important.

Fejér was the founder of the Hungarian school of analysis. A remarkable group of analysts came through this school. George Pólya wrote of Gabor Szegő, "We were both influenced, like all other young Hungarian mathematicians of that time, by Leopold Fejér." This was about 1910 to 1915. Twenty years later Turán was also heavily influenced by Fejér. He remarks on this in the lecture he gave at the 70th anniversary meeting of the births of Fejér and Frederick Riesz. To show the style that Turán used, after commenting that Fejér's refined techniques would often do in a few lines what others did in pages, he remarked that to use the same word "calculation" for both was analogous to the use in Hungarian of the same word for the clumsy hopping of a bear and a ballerina's pirouettes.

I have used the Fejér papers often. Turán's editing was remarkable. He commented on many of the papers, setting them into context and telling what has happened to the ideas Fejér introduced. For example, the comments on Fejér's great work on $(C, 1)$ summability on Fourier series include references from H. Lebesgue's 1906 book to a paper on abstract ergodic theorems by W. F. Ebelein in 1949. One interesting remark which I had not known is that Fejér's name was misspelled "Tejér" in both the journal and in the review in *Fortschritte der Mathematik*.

Another instance of the care that Turán took is given in his "Das Leben von Leopold Fejér" which starts the first volume. He quotes Hardy about this result of Fejér as follows: "...this fundamental result has been the starting point of a mass of modern research." Turán's citation of this is from

the 1922 “Encyclopaedia Britannica.” The 11th edition of the Britannica from 1910–1911 is justly famous. The 1922 edition is the 12th, which consists of 29 volumes of the 11th and 3 added volumes. The 3 new volumes are indexed separately and so have not been consulted nearly as much as the first 29 volumes. G. H. Hardy’s article is in the 30th volume. There is a reference to Fejér in the index of these 3 volumes, and that Turán thought to look there shows the scholar he could be when he turned his mind to it.

This scholarly side of Turán also showed itself in his knowledge of the history of mathematics. He could tell stories about the history of mathematics seemingly without end. However, much of his own mathematics was done without his having searched the literature. He had many original ideas, and they were frequently all he needed.

The Fejér papers were published in 1970. Shortly thereafter, I started to look seriously at some positive sums of the classical orthogonal polynomials. I used papers of Szegő and the Fejér papers as my main resources, to see what had been done and to suggest problems. Within 5 years, George Gasper had worked out a proof of a conjecture of mine, and 10 years later a special case was used by Louis de Branges as the final step in his solution of the Bieberbach conjecture. While it is probably too strong a statement to say that this work would not have been done without the ready access to Fejér’s papers, the work would have been done more slowly.

Another time during which Turán’s editing had an impact on me was when I edited Gabor Szegő’s “Collected Papers.” I had Turán’s model on which to build and used it when writing comments on Szegő’s papers.

It is likely that similar comments will eventually be made about the papers of Rényi which Turán edited and of Turán’s papers, for collected papers of great mathematicians are one of the best ways of passing on original ideas.

B. Meulenbeld told me a story about Turán which illustrates Turán’s playfulness. They were in the same compartment of a train, but did not know each other. After some pleasant words, they discovered they were both mathematicians. After Meulenbeld introduced himself and said a little about what he did, it was Turán’s turn to do the same. He took out a piece of paper and wrote $P_n^2(x) - P_{n+1}(x)P_{n-1}(x) \geq 0$, and asked Meulenbeld if he knew this. Meulenbeld said, “Of course, that is Turán’s inequality.” Turán then said “I am Turán.”

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